Process and Product Improvement in Manufacturing Systems with Correlated Stages

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Manufacturing systems typically contain processing and assembly stages whose output quality is significantly affected by the output quality of preceding stages in the system. This study offers and empirically validates a procedure for (1) measuring the effect of each stage's performance on the output quality of subsequent stages including the quality of the final product, and (2) identifying stages in a manufacturing system where management should concentrate investments in process quality improvement. Our proposed procedure builds on the precedence ordering of the stages in the system and uses the information provided by correlations between the product quality measurements across stages.

The starting point of our procedure is a *computer executable* network representation of the statistical relationships between the product quality measurements; execution automatically converts the network to a simultaneous-equations model and estimates the model parameters by the method of least squares. The parameter estimates are used to measure and rank the impact of each stage's performance on variability in intermediate stage and final product quality. We extend our work by presenting an economic model, which uses these results, to guide management in deciding on the amount of investment in process quality improvement for each stage.

We report some of the findings from an extensive empirical validation of our procedure using circuit board production line data from a major electronics manufacturer. The empirical evidence presented here highlights the importance of accounting for quality linkages across stages in (a) identifying the sources of variation in product quality and (b) allocating investments in process quality improvement.

(*Quality and Process Improvement; Total Quality Management; Investments in Learning; Multistage Manufacturing Systems*)

1. Introduction

The use of statistical quality control and related quality-improvement methods has become widespread in recent years as a result of the increased emphasis on improving quality and product competitiveness. An important premise underlying these methods is that reducing process and product variability leads to improved products and reduced quality costs. Reducing variability is also known to favorably affect operating metrics such as productivity, cycle time, and capacity. Tagaras and Lee (1996) and Williams and Peters (1989) recognize that the output quality of some stages in multistage manufacturing systems is significantly affected by the

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output quality of preceding stages. Most of the literature on quality control and improvement, however, is restricted to single-stage models or assumes the absence of quality linkages across stages. A notable exception is Hawkins (1993), who proposes a procedure for monitoring process quality in manufacturing systems where the measures of output quality are correlated across stages.

This study proposes and validates a procedure for *measuring the effect* of each stage's performance on the output quality of subsequent stages, including the quality of the final product. The proposed procedure builds on the precedence ordering of the stages in the system and uses the information provided by correlations between the product quality measurements across stages. The starting point of the procedure is a *computer executable* network representation of the statistical relationships between the product quality measurements; execution automatically converts the network to a simultaneous-equations model and estimates the model parameters by the method of least squares.[1] The parameter estimates are used to measure and rank the impact of each stage's performance on variability in intermediate stage and final product quality.

The model and procedure developed in this paper are applied to circuit board production line data from a major electronics manufacturer. The data set used in this empirical validation includes measurements on several machine performance and product quality variables from both the intermediate and final stages. We illustrate how the procedure can guide management in deciding how quality-improvement resources should be allocated among the stages of the manufacturing system. Our empirical findings demonstrate that quality linkages across stages need to be accounted for in identifying the sources of variability in product quality. Moreover, our results imply that ignoring quality linkages can lead manufacturers to make suboptimal investments in quality improvement. Thus, our study calls attention to the need for manufacturers to incorporate information on quality linkages into decisions about quality-improvement investments. In terms of implications for statistical process control, we present empirical evidence that conventional process-monitoring procedures (statistical process control charts) are not appropriate for identifying stages that are out of control. This emphasizes the need for manufacturers to use process-monitoring procedures that are designed for multistage manufacturing systems.

Decisions about investments in process quality improvement have the potential to favorably impact quality-related costs (e.g., the costs associated with producing nonconforming products) and are crucial to the efficacy and success of quality-improvement initiatives. By augmenting our multistage model with research on continuous quality improvement and learning, we extend our work to formally incorporate quality costs into decision making about investments in quality improvement. To our knowledge, previous studies on investments in quality improvement and learning are restricted to single-stage models. We develop an economic model that links investments in quality improvement to reduced variation in product quality and, in turn, savings in quality costs. The economic model specifies that investments in quality improvement face diminishing marginal returns with respect to savings in quality costs. The model is used to derive the optimal amount of investment in quality improvement for each stage of the manufacturing system.

Our study makes three important contributions to the literature. First, it offers and empirically validates a model and procedure for measuring the impact of each stage's performance on variation in intermediate and final product quality. Knowledge of each stage's impact on product variability is useful in evaluating stage performance and in allocating quality-improvement resources. The second contribution of our research is that we present an economic approach for deciding on the amount of investment in process quality improvement for each stage of the manufacturing system. Finally, we demonstrate the importance of identifying and measuring quality linkages across stages in quality-improvement initiatives such as statistical process control and Total Quality Management (TQM) programs.

[1] The network execution software is written by Wynne Chin and Timothy Frye at the University of Calgary, Alberta, Canada.

1.1. Related Literature

Two distinct streams of research address quality control and quality improvement for multistage manufacturing systems. Within the operations management literature, several papers examine the allocation of inspection effort in multistage manufacturing systems. Lindsay and Bishop (1964), Eppen and Hurst (1974), Ballou and Pazer (1982), Chevalier and Wein (1997), and others (see Raz 1986 for a review) consider the optimal allocation of inspection stations and level of inspection at each station. Ballou and Pazer (1985) examine the optimal combination of inspection station placement, inspection process improvement, and manufacturing process improvement. Within the statistical process control literature, the papers by Hawkins (1993) and Williams and Peters (1989) study the design of procedures for monitoring process quality in manufacturing systems where the measures of output quality are correlated across stages. In contrast to our study, however, the above papers are not concerned with measuring each stage's impact on variation in product quality or determining the amount of investment in process quality improvement for each stage.

The engineering process control literature (see, e.g., Montgomery 1996, pp. 386–394 and Janakiram and Keats 1998 and references cited therein) is also related to our study in that it has reduction of product variability as its objective. Applied primarily in continuous processes where the product quality variables are autocorrelated, engineering process control attempts to maintain the product quality variable at a target value through regular adjustment of a process input. Thus, engineering process control attempts to reduce predictable quality variation whereas the focus of our study is on identifying (a) the sources of variation in product quality and (b) the optimal amount of investment in process quality improvement for each stage of the manufacturing system.

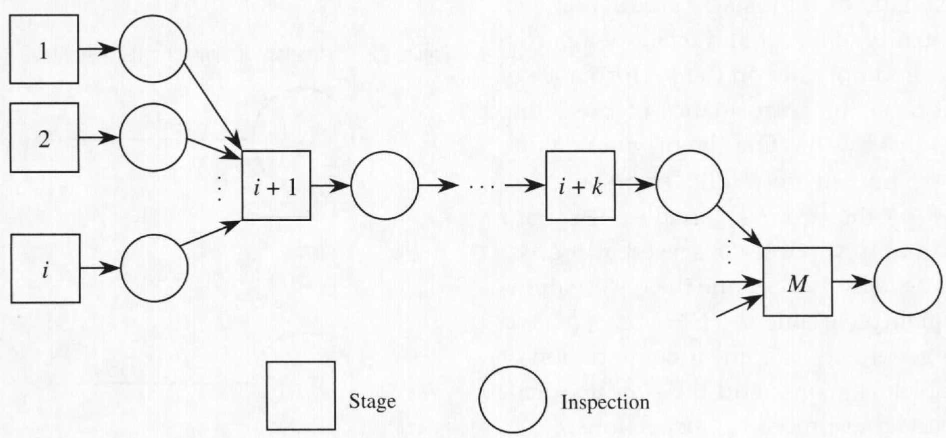
1.2. Organization of the Paper

The remainder of the paper is organized as follows. Section 2 gives our variable definitions and presents a simultaneous-equations model of the statistical relationships between the stages of a general acyclic manufacturing system. Section 3 describes our procedure and focuses on model parameter estimation. In §4, we discuss an application of our model and procedure to circuit board production line data. Section 5 develops an economic approach for deciding on the amount of investment in process quality improvement for each stage. Section 6 closes with a discussion of research issues and implications for quality-improvement practice.

2. The Model

Consider an acyclic manufacturing system comprised of M stages. Suppose, as shown in Figure 1, the stages are numbered in ascending order such that if stage i precedes stage k, then $i < k$. The flow of each unit u through the system can be described as follows. At

Figure 1 **Acyclic Manufacturing System**

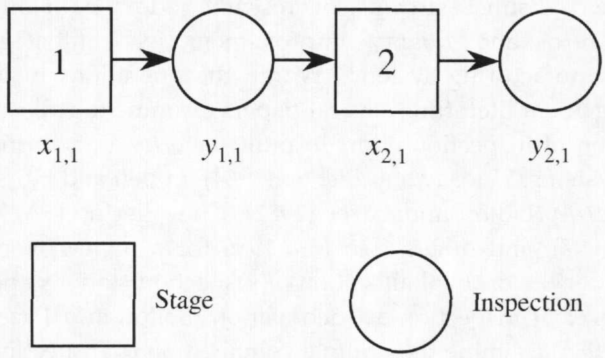


each stage i $(=1,2,\ldots,M)$ one or more operations (assembly, conversion, fabrication) are performed on the unit. Measurements from these processes (e.g., temperature, pressure, etc.) may be observed. Suppressing the unit identifier, let $x_{i,j}$ denote the deviation of the jth *process variable* observed at stage i from its mean. We assume that process variables observed at different stages are statistically independent, i.e., $x_{i,j}$ is independent of $x_{k,l}$ for $i \neq k$; process variables at the same stage, however, may be correlated, i.e., $x_{i,j}$ may correlate with $x_{k,l}$ only if $i=k$. Denote the number of process variables observed at stage i by p_i, and let $p=\sum_{i=1}^{M} p_i$ denote the total number of process variables observed in the system. Upon completion of each stage, a product inspection is performed where one or more quality measurements are taken from the unit. Let $y_{i,j}$ denote the deviation of the jth *product quality variable* observed at the inspection following stage i $(=1,\ldots,M)$ from its mean. We assume $y_{i,j}$ is a continuous random variable for all i and j. Denote the number of product quality variables observed at the inspection following stage i by q_i, and let $q=\sum_{i=1}^{M} q_i$ denote the total number of product quality variables. After the inspection is performed, the unit proceeds to the subsequent stage. We initially assume that (a) no rework is performed and (b) the unit does not leave the system before the final inspection (the inspection following stage M). In §4, we describe a straightforward modification that allows for rework of nonconforming units.

2.1. Specification

A salient characteristic of multistage production systems is that the quality of a unit at the end of a particular stage can depend not only on the performance of that stage but also on the performance of preceding stages. We model this by allowing the product quality variables observed immediately following stage i to be *directly affected* by the process variables at stage i and the product quality variables observed at preceding inspections. Formally, we assume the observations on the product quality variable $y_{i,j}$ $(i=1,\ldots,M$ and $j=1,\ldots,q_i)$ are generated as a linear combination of the process variables at stage i and the product quality variables observed at preceding inspections.

Figure 2A Two-Stage Serial System



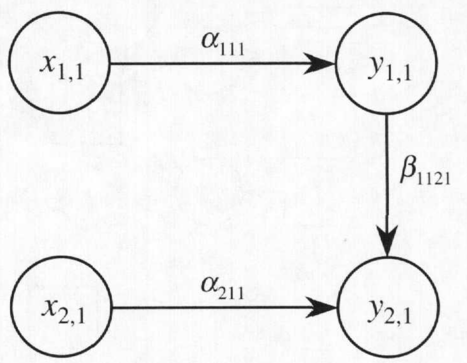
To illustrate our model specification, consider the two-stage serial system in Figure 2A. For illustration, we consider the simple case in which one process variable is observed at each stage and one product quality variable is observed immediately following each stage, i.e., $p_1=p_2=q_1=q_2=1$. The statistical model for this system, in network form, is given in Figure 2B. This model is written in simultaneous-equations form as

$$y_{1,1}=\alpha_{111}x_{1,1}+\varepsilon_{11} \tag{1}$$

$$y_{2,1}=\alpha_{211}x_{2,1}+\beta_{1121}y_{1,1}+\varepsilon_{21}, \tag{2}$$

where the coefficients α_{111}, α_{211}, and β_{1121} are parameters to be estimated and ε_{11} and ε_{21} are random error terms with zero mean. (A constant term is absent from (1) and (2) because the variables $x_{i,j}$ and $y_{i,j}$ have zero mean.) Equation (1) specifies that the product quality variable $y_{1,1}$ observed following

Figure 2B Statistical Model for the Two-Stage Serial System



Stage 1 may be directly affected by $x_{1,1}$, the process variable at Stage 1. Equation (2) specifies that the product quality variable $y_{2,1}$ observed following Stage 2 may be directly affected by the process variable at Stage 2 ($x_{2,1}$) and the product quality variable observed immediately following Stage 1 ($y_{1,1}$).

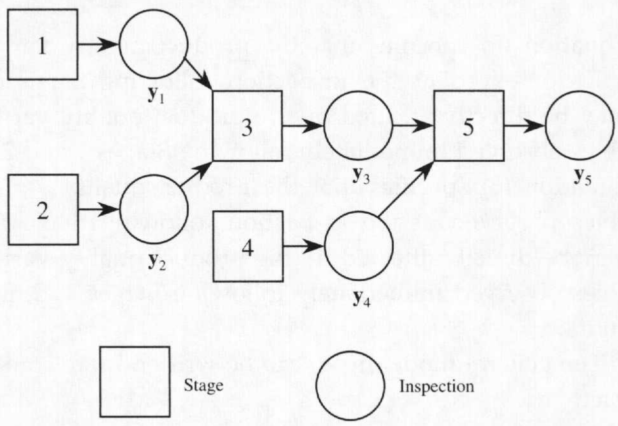
To present a more general formulation of our model, we introduce the following notation. Let the parameter α_{ilj} (to be estimated) denote the direct effect of the lth process variable observed at stage i on the jth product quality variable observed at the inspection following stage i. For each stage i, let D_i denote the set of its predecessors. Let the parameter β_{klij} (to be estimated) denote the direct effect of the lth product quality variable observed immediately following stage k on the jth product quality variable observed immediately following stage i, $k \in D_i$. These definitions enable us to write the general formulation of our model as

$$y_{i,j} = \sum_{l=1}^{p_i} \alpha_{ilj} x_{i,l} + \sum_{k \in D_i} \sum_{l=1}^{q_k} \beta_{klij} y_{k,l} + \varepsilon_{ij}, \quad i = 1, \dots, M \quad \text{and} \quad j = 1, \dots, q_i, \qquad (3)$$

where ε_{ij} is a random error term with mean zero and variance σ_{ij}^2. The simultaneous-equations model in (3) expresses the product quality variable $y_{i,j}$ as a linear combination of (a) the process variables observed at stage i ($\sum_{l=1}^{p_i} \alpha_{ilj} x_{i,l}$), (b) the product quality variables observed at preceding inspections ($\sum_{k \in D_i} \sum_{l=1}^{q_k} \beta_{klij} y_{k,l}$), and (c) a random error term (ε_{ij}). The error term ε_{ij} represents the net effect of unobserved process variables at stage i on $y_{i,j}$. We assume that ε_{ij} is distributed independently of $x_{k,l}$ for all i, j, k, and l. We further assume that ε_{ij} is independent of ε_{kl} for $i \neq k$, i.e., the ε_{ij} are uncorrelated across stages.

To facilitate subsequent discussion, we now set out as compactly as possible the simultaneous-equations model in (3) using vector-matrix notation. Let the vector $(y_{i,1}, y_{i,2}, \dots, y_{i,q_i}) = \mathbf{y}_i$ be the $1 \times q_i$ vector of product quality variables observed at the inspection following stage i ($= 1, \dots, M$). Similarly, let the vector $(x_{i,1}, x_{i,2}, \dots, x_{i,p_i}) = \mathbf{x}_i$ be the $1 \times p_i$ vector of process variables observed at stage i. These definitions enable

Figure 3A Assembly System



us to write the system of equations in (3) compactly in vector-matrix form as

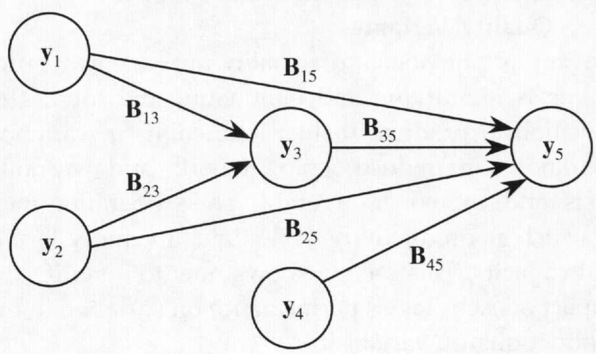
$$\mathbf{y}_i = \mathbf{x}_i \mathbf{A}_i + \sum_{k \in D_i} \mathbf{y}_k \mathbf{B}_{ki} + \boldsymbol{\varepsilon}_i \qquad (i = 1, \dots, M), \qquad (4)$$

where $\mathbf{A}_i$ is a $p_i \times q_i$ matrix of coefficients with ljth entry α_{ilj}, $\mathbf{B}_{ki}$ is a $q_k \times q_i$ matrix of coefficients with ljth element β_{klij}, and $\boldsymbol{\varepsilon}_i$ is a $1 \times q_i$ vector of random errors with ε_{ij} in the jth position. Denote the $q_i \times q_i$ variance-covariance matrix of $\boldsymbol{\varepsilon}_i$ by Σ_i, $i = 1, \dots, M$.

Figure 3A is an example of an $M = 5$ stage assembly system. Stage 3 assembles the outputs of Stages 1 and 2, and Stage 5 assembles the outputs of Stages 3 and 4. (This example assumes, for ease in exposition, the absence of observable process variables in the system.) The model for this system is given in Figure 3B and is written in simultaneous-equations form as

$$\mathbf{y}_3 = \mathbf{y}_1 \mathbf{B}_{13} + \mathbf{y}_2 \mathbf{B}_{23} + \boldsymbol{\varepsilon}_3 \qquad (5)$$

Figure 3B Simultaneous-Equations Model for the Assembly System



$$\mathbf{y}_5 = \mathbf{y}_1\mathbf{B}_{15} + \mathbf{y}_2\mathbf{B}_{25} + \mathbf{y}_3\mathbf{B}_{35} + \mathbf{y}_4\mathbf{B}_{45} + \boldsymbol{\varepsilon}_5. \quad (6)$$

Equation (5) specifies that the product quality variables observed at the inspection following Stage 3 may be directly affected by the product quality variables observed immediately following Stages 1 and 2. Equation (6) specifies that the product quality variables observed at the inspection following Stage 5 may be directly affected by the product quality variables observed immediately following Stages 1, 2, 3, and 4.

The general model in (4) can be written more compactly as

$$\mathbf{y} = \mathbf{x}\mathbf{A} + \mathbf{y}\mathbf{B} + \boldsymbol{\varepsilon}, \quad (7)$$

where $\mathbf{y} = (\mathbf{y}_1, \ldots, \mathbf{y}_M)$, $\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_M)$, $\boldsymbol{\varepsilon} = (\boldsymbol{\varepsilon}_1, \ldots, \boldsymbol{\varepsilon}_M)$, $\mathbf{A}$ is a $p \times q$ matrix of coefficients whose ikth submatrix equals $\mathbf{A}_i$ if $i = k$ and equals $\mathbf{0}$ otherwise, where $\mathbf{0}$ is a $p_i \times q_k$ matrix of zeros, and $\mathbf{B}$ is a $q \times q$ matrix of coefficients whose ikth submatrix equals $\mathbf{B}_{ik}$ if $i \in D_k$ and equals $\mathbf{0}$ otherwise, where $\mathbf{0}$ is a $q_i \times q_k$ matrix of zeros. The error vector $\boldsymbol{\varepsilon}$ in (7) has zero mean and $q \times q$ block-diagonal variance-covariance matrix

$$\boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_1 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_2 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\Sigma}_3 & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{\Sigma}_M \end{pmatrix}. \quad (8)$$

The ikth submatrix of $\boldsymbol{\Sigma}$ in (8) equals Σ_i if $i = k$ and equals $\mathbf{0}$ otherwise, where $\mathbf{0}$ is a $q_i \times q_k$ matrix of zeros.

2.2. Sources of Variation in the Product Quality Variables

An important focus of quality-improvement programs is identifying and eliminating the sources of variation in product quality. Reductions in variation are known to reduce scrap, rework, and warranty costs, and, moreover, favorably affect operating metrics such as productivity, cycle time, inventory levels, and capacity. This section shows how to quantify the impact of each stage's performance on variation in the product quality variables.

First, subtract $\mathbf{yB}$ from both sides of (7) to get

$$\mathbf{y}(\mathbf{I}_q - \mathbf{B}) = \mathbf{x}\mathbf{A} + \boldsymbol{\varepsilon}, \quad (9)$$

where $\mathbf{I}_q$ denotes the $q \times q$ identity matrix. Postmultiplying both sides of the preceding expression by $\mathbf{C} = (\mathbf{I}_q - \mathbf{B})^{-1}$ gives

$$\mathbf{y} = \mathbf{x}\mathbf{A}\mathbf{C} + \boldsymbol{\varepsilon}\mathbf{C}. \quad (10)$$

Then, taking the variance on both sides of (10) and using our assumption that the process variables in $\mathbf{x}$ are uncorrelated with the error terms in $\boldsymbol{\varepsilon}$, we obtain

$$\mathrm{Var}(\mathbf{y}) = \mathbf{C}'\mathbf{A}'\,\mathrm{Var}(\mathbf{x})\mathbf{A}\mathbf{C} + \mathbf{C}'\boldsymbol{\Sigma}\mathbf{C}, \quad (11)$$

where $\mathrm{Var}(\cdot)$ denotes the variance-covariance matrix of $(\cdot)$. Equation (11) decomposes the variance of each product quality variable into components attributable to each stage.[2]

To demonstrate how the decomposition in (11) is used to compute the contribution of each stage's performance to variation in product quality, consider the model in Figure 2B. Recall that this model is given in equation form in (1) and (2). Application of (11) to this model decomposes the variance of the product quality variable $y_{1,1}$ as

$$\mathrm{Var}(y_{1,1}) = \alpha_{111}^2\,\mathrm{Var}(x_{1,1}) + \sigma_{11}^2, \quad (12)$$

where σ_{11}^2 denotes the variance of ε_{11}. The first term (variance component) on the right-hand side of (12) represents the amount of variation in $y_{1,1}$ due to the process variable $x_{1,1}$. The second variance component

[2] The formula in Equation (11) assumes all product quality variables in the model (7) are endogenous (i.e., determined by variables within the model). To modify (11) for models that contain exogenous product quality variables (i.e., product quality variables whose determinants lie outside the model), let the row vector $\mathbf{x}^*$ contain the process variables and the exogenous product quality variables. Next, let the row vector $\mathbf{y}^*$ contain the endogenous product quality variables. We can now rewrite Equation (7) as $\mathbf{y}^* = \mathbf{x}^*\mathbf{A}^* + \mathbf{y}^*\mathbf{B}^* + \boldsymbol{\varepsilon}^*$, where $\mathbf{A}^*$ and $\mathbf{B}^*$ are appropriately-defined matrices of coefficients and $\boldsymbol{\varepsilon}^*$ is a row vector of errors with mean zero and variance-covariance matrix $\boldsymbol{\Sigma}^*$. It follows immediately from (7) and (11) that the variance-covariance matrix of $\mathbf{y}^*$ can be decomposed as $\mathrm{Var}(\mathbf{y}^*) = \mathbf{C}^{*\prime}\mathbf{A}^{*\prime}\,\mathrm{Var}(\mathbf{x}^*)\mathbf{A}^*\mathbf{C}^* + \mathbf{C}^{*\prime}\boldsymbol{\Sigma}^*\mathbf{C}^*$, where $\mathbf{C}^* = (\mathbf{I} - \mathbf{B}^*)^{-1}$. This formula is used to compute the impact of each stage's performance on variation in the product quality variables.

on the right-hand side represents the amount of variation in $y_{1,1}$ due to unobserved process variables at Stage 1. Application of (11) to the model in Figure 2B decomposes the variance of the product quality variable $y_{2,1}$ as

$$\mathrm{Var}(y_{2,1}) = \alpha_{111}^2\beta_{1121}^2 \mathrm{Var}(x_{1,1}) + \alpha_{211}^2 \mathrm{Var}(x_{2,1}) + \beta_{1121}^2\sigma_{11}^2 + \sigma_{21}^2, \quad (13)$$

where σ_{21}^2 denotes the variance of ε_{21}. The first term on the right-hand side of (13) represents the amount of variation in $y_{2,1}$ due to the process variable $x_{1,1}$. Similarly, the second term represents the amount of variation in $y_{2,1}$ due to the process variable $x_{2,1}$. The third and fourth terms represent the amount of variation in $y_{2,1}$ due to unobserved process variables at Stages 1 and 2, respectively. Hence, in this example we may view the variance of the product quality variable $y_{2,1}$ as the sum of the variance due to sources at Stage 1, and the variance due to sources at Stage 2.

3. Procedure

This section describes the procedure for measuring the effect of each stage's performance on the output quality of subsequent stages. Section 3.1 discusses the estimation of the parameter matrices **A**, **B**, and **Σ**. Section 3.2 deals with estimating the impact of each stage's performance on variation in product quality.

3.1. Estimation of Parameters

Here we discuss the estimation of the parameter matrices **A**, **B**, and **Σ** from a data sample of N observations on the p process variables and q product quality variables. It follows from the results in §3 of Zantek (2000) that the parameters of our simultaneous-equations model are uniquely identified. From the fact that the matrix **B** in (7) is upper triangular and our assumptions that (a) ε_{ij} is independent of ε_{kl} for $i \neq k$ and (b) ε_{ij} is independent of $x_{k,l}$ for all i, j, k, and l, it follows that $y_{k,l}$ is uncorrelated with ε_{ij} for $k \in D_i$. Therefore, all the variables on the right-hand side of (3) are uncorrelated with the error term (ε_{ij}) and application of ordinary least squares to (3) will yield unbiased and consistent estimates of the coefficients α_{ilj} and β_{klij}.[3]

Let $\widehat{\mathbf{A}}_i$ and $\widehat{\mathbf{B}}_{ik}$ denote the matrices obtained by substituting in the obvious manner the estimated coefficients into $\mathbf{A}_i$ and $\mathbf{B}_{ik}$, respectively. Then, for each stage i $(=1,2,\ldots,M)$, define the $N \times q_i$ matrix of residuals $\mathbf{E}_i = \mathbf{Y}_i - \widehat{\mathbf{Y}}_i$, where $\mathbf{Y}_i$ is the $N \times q_i$ data matrix of observations on the product quality variables in $\mathbf{y}_i$ and

$$\widehat{\mathbf{Y}}_i = \mathbf{X}_i\widehat{\mathbf{A}}_i + \sum_{k\in D_i} \mathbf{Y}_k\widehat{\mathbf{B}}_{ki}, \quad (14)$$

where $\mathbf{X}_i$ is the $N \times p_i$ data matrix of observations on the process variables in $\mathbf{x}_i$. A consistent estimator of $\boldsymbol{\Sigma}$ in (8), is

$$\widehat{\boldsymbol{\Sigma}} = \begin{pmatrix} \widehat{\boldsymbol{\Sigma}}_1 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \widehat{\boldsymbol{\Sigma}}_2 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \widehat{\boldsymbol{\Sigma}}_3 & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \widehat{\boldsymbol{\Sigma}}_M \end{pmatrix}, \quad (15)$$

where

$$\widehat{\boldsymbol{\Sigma}}_i = \mathbf{E}_i'\mathbf{E}_i/N. \quad (16)$$

3.2. Estimation of Variance Components

To quantify the impact of each stage's performance on variation in the product quality variables, first estimate the $p \times p$ variance-covariance matrix of the process variables, $\mathrm{Var}(\mathbf{x})$, by

$$\mathbf{V} = \begin{pmatrix} \mathbf{V}_1 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_2 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{V}_3 & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{V}_M \end{pmatrix}, \quad (17)$$

where

$$\mathbf{V}_i = \frac{1}{N}\mathbf{X}_i'(\mathbf{I}_N - \mathbf{1}\mathbf{1}'/N)\mathbf{X}_i \qquad (i=1,2,\ldots,M), \quad (18)$$

where $\mathbf{1}$ is an $N \times 1$ vector of ones. Let $\widehat{\mathbf{A}}$ and $\widehat{\mathbf{B}}$ denote the matrices obtained by substituting the estimated

[3] In the case where $\boldsymbol{\varepsilon}$ is a multivariate normal random vector, it follows from §3 of Zantek (2000) that the ordinary least squares estimates of α_{ilj} and β_{klij} are identical to the maximum likelihood estimates.

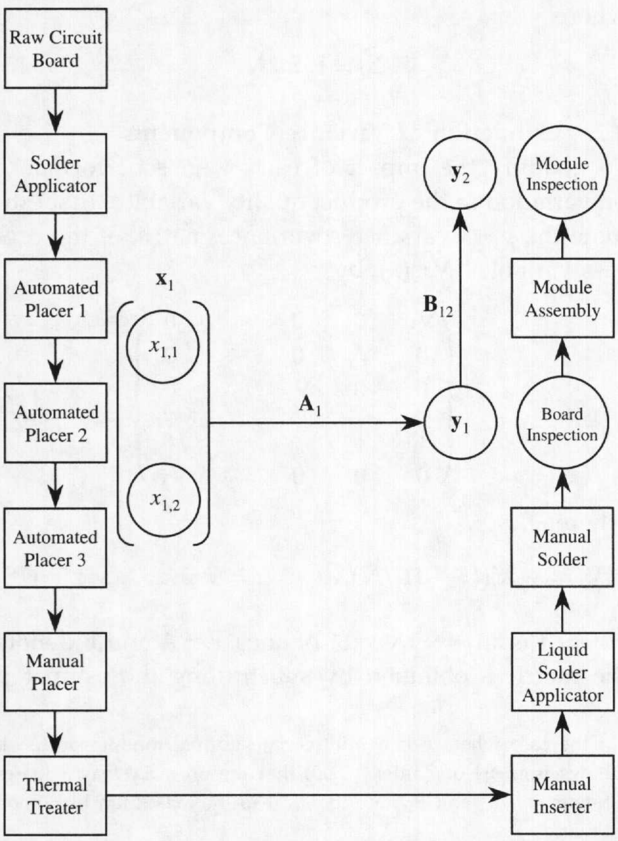
coefficients into **A** and **B**, respectively. Then, estimate the variance components by using (11) and the estimated parameter matrices $\widehat{\mathbf{A}}$, $\widehat{\mathbf{B}}$, $\widehat{\mathbf{\Sigma}}$, and **V**. Since $\widehat{\mathbf{A}}$, $\widehat{\mathbf{B}}$, $\widehat{\mathbf{\Sigma}}$, and **V** are consistent estimators and the variance components are continuous functions of the elements of **A**, **B**, **Σ**, and Var(**x**), it follows from the Slutsky theorem (see, e.g., Greene 2000, pp. 112–113) that this procedure yields consistent estimates of the variance components. (For the case where **x** and **ε** are multivariate normal random vectors, it can be shown that this procedure yields the maximum likelihood estimates of the variance components.)

4. Empirical Application

4.1. Process Description

We begin by describing the circuit board assembly and inspection process, which is depicted in Figure 4. For our purposes, we view the process as consisting of $M = 2$ stages: Board Assembly and Module Assembly. Board Assembly consists of nine stations, beginning at Solder Applicator and ending at Manual Solder. At Solder Applicator, a solder paste is applied to each circuit board. Next, electronic components are placed on the circuit board by automated placer machines at the stations Automated Placer 1, 2, and 3. Components requiring manual placement are placed on the circuit board by an assembly worker at Manual Placer. Thermal Treater consists of heating and cooling operations where the solder paste applied at Solder Applicator is melted and subsequently cooled, causing the components placed at previous stations (Automated Placer 1, 2, and 3) to be joined to the board. At the following station, Manual Inserter, an assembly worker inserts *through-hole* components into predrilled holes in the board. To solder these components to the board, the board is next immersed in liquid solder. Electronic components requiring manual soldering are soldered to the board at Manual Solder, the final station in Board Assembly.

Figure 4 Circuit Board Production Line and Model Specification



At Board Inspection, the board undergoes extensive testing and numerous product quality measurements are taken from the board. The board is reworked by technicians if one or more of these measurements falls outside specifications. At Module Assembly, each board is combined with a circuit board assembled on another line, yielding what is called a *module*. Comprehensive tests are performed on the module at Module Inspection and approximately 200 product quality measurements are taken. The module is reworked by technicians if one or more of these measurements falls outside specifications.

4.2. Data

The data come from several real-time databases maintained by the manufacturer. These databases track each board and record numerous process and product quality measurements. The databases contain 11 product quality measurements from Board Inspection. In addition, of the approximately 200 product quality measurements taken at Module Inspection, 30 measurements deemed important by management and engineering are stored in the databases. Finally, the databases contain two machine performance measurements from each of the three automated placer machines.[4] Each measurement in the databases is associated with a serial number that uniquely identifies the circuit board to which it applies.

From the above databases, we construct a single data set that consists of the measurements from Board Inspection, Module Inspection, and the three automated placer machines for a two-week period. In the first step of the data cleaning, we eliminate the records corresponding to circuit boards for which we do not have measurements from all stations; this leaves 115 records. We next clean the data by deleting records that contain missing values. The remaining $N = 89$ records comprise the sample from which we estimate the parameters of the simultaneous-equations model (3). Each record corresponds to an individual circuit board and contains measurements on the 11 product quality variables observed at Board Inspection and the 30 product quality variables observed at Module Inspection. Also accompanying each record are measurements on the two-machine performance (i.e., process) variables from each of the three automated placer machines. All variables are scaled to zero mean and unit variance in order to preserve the confidentiality of the data.

4.3. Data Reduction

The six process variables from the automated placer machines are highly intercorrelated. Therefore, prior to applying the procedure in §3, we factor analyze the correlation matrix of these six variables, using the principal factor method. Table 1 presents the results for a two-factor solution. Examining the loadings in Table 1, we conclude that Factor 1 represents the performance of Automated Placers 1 and 2; Factor 2 represents the performance of Automated Placer 3. We also see from Table 1 that the two factors collectively account for 100% of the variation in the six observed process variables. Finally, we note from the loadings in the table that each of the six process variables is highly correlated with one of the two factors, implying that the two factors are good proxies for the original variables. We therefore reduce the six process variables to two factors. The observations on the two factors are computed using the regression

[4] No process variables are observed at the Module Assembly stage.

Table 1 Results from the Factor Analysis of the Six Automated Placer Variables

Observed Process Variable	Loadings* Factor 1	Loadings* Factor 2
Automated Placer 1 Variable 1	0.99	0.05
Automated Placer 1 Variable 2	0.99	0.03
Automated Placer 2 Variable 1	0.97	0.26
Automated Placer 2 Variable 2	0.89	−0.46
Automated Placer 3 Variable 1	0.03	0.99
Automated Placer 3 Variable 2	0.03	0.99
	Factor 1	Factor 2
Variance Explained†	3.72	2.28
% of Variance Explained	62.06	37.94
Cumulative % of Variance Explained	62.06	100.00

*The loadings obtained by the principal factor method are transformed by a varimax rotation.

†Total variance of the six process variables accounted for by the factor. The total variance of the six process variables equals 6 since the variables are standardized.

approach (see Anderson 1984, pp. 575–576, or Johnson and Wichern 1998, p. 553). In the following we use the two factors as the process variables $x_{1,1}$ and $x_{1,2}$ of the simultaneous-equations model (3).

4.4. Estimation Results

The simultaneous-equations model for the circuit board assembly system is set out in Figure 4. Table 2 presents the estimation results for selected model equations. The estimate of the $y_{1,2}$ equation, for example, is

$$y_{1,2} = 0.31x_{1,1} + 0.19x_{1,2} + \hat{\varepsilon}_{1,2}. \qquad (19)$$

For each model coefficient, we apply a two-sided t-test to check if the coefficient is statistically different from zero; the asterisks in Table 2 indicate those coefficients determined to be statistically different from zero (i.e., statistically significant) at the 1% level of significance (two asterisks) and the 5% level of significance (one asterisk). From Table 2, we see the process variables $x_{1,1}$ and $x_{1,2}$ have strong and statistically significant effects on $y_{1,2}$ and $y_{1,4}$ and $y_{1,4}$ respectively. For example, the effect of $x_{1,1}$ on $y_{1,2}$ is 0.31, which is statistically significant at the 1% level. Since the coefficients are standardized, this indicates

Table 2 Selected Estimation Results

Predictor Variable	Equation									
	$y_{1,1}$	$y_{1,2}$	$y_{1,4}$	$y_{1,11}$	$y_{2,1}$	$y_{2,2}$	$y_{2,7}$	$y_{2,13}$	$y_{2,28}$	$y_{2,29}$
$x_{1,1}$	0.12	0.31**	0.54**	−0.02	—	—	—	—	—	—
$x_{1,2}$	−0.10	0.19	0.31**	−0.08	—	—	—	—	—	—
$y_{1,1}$	—	—	—	—	−0.05	0.05	0.08*	−0.05	−0.09	0.01
$y_{1,2}$	—	—	—	—	−0.05	−0.08	−0.08*	−0.01	−0.05	−0.00
$y_{1,3}$	—	—	—	—	−0.18	0.04	0.06	0.23	−0.16	0.04
$y_{1,4}$	—	—	—	—	0.19	−0.07	−0.05	−0.06	−0.02	−0.05
$y_{1,5}$	—	—	—	—	−0.18	0.08	0.04	0.08	−0.13	0.01
$y_{1,6}$	—	—	—	—	−0.07	0.05	−0.04	−0.02	0.05	0.04
$y_{1,7}$	—	—	—	—	0.78**	0.46**	−0.93**	−0.12	0.08	0.11
$y_{1,8}$	—	—	—	—	−0.06	0.09	0.13**	0.10	−0.11	−0.15*
$y_{1,9}$	—	—	—	—	0.03	0.04	−0.03	0.03	−0.01	0.60**
$y_{1,10}$	—	—	—	—	0.05	−0.01	0.01	−0.04	0.65**	0.12
$y_{1,11}$	—	—	—	—	0.03	0.01	−0.03	0.05	−0.01	−0.05
R-Square	0.02	0.13**	0.39**	0.01	0.70**	0.27**	0.81**	0.05	0.41**	0.47**

*Statistically significant at the 5% level.
**Statistically significant at the 1% level.

that a one standard deviation increase in $x_{1,1}$ leads to a 0.31 standard deviation increase in $y_{1,2}$. Another notable finding in Table 2 is the significant correlation between the output quality of the Board Assembly and Module Assembly stages. The R-squared statistics and coefficient estimates in Table 2 provide evidence that the Board Assembly stage substantially affects the output quality of the Module Assembly stage. We also note from the R-squared statistics in Table 2 that the Board Inspection variables account for most of the variation in the Module Inspection variables $y_{2,1}$ and $y_{2,7}$; this implies that a majority of the variation in $y_{2,1}$ and $y_{2,7}$ is due to the Board Assembly stage.

Overall, the estimation results support our model specification. First, as discussed previously, the results in Table 2 strongly suggest that the Board Assembly stage affects the output quality of the Module Assembly stage. Second, with only a few exceptions, examination of the residuals of each model equation does not suggest heteroscedasticity, model specification errors, or any significant outliers in the data.

Having established that our model provides a good fit to the data, we use the procedure outlined in §3.2 to decompose the variance of each product quality variable into components. The estimated variance components of selected product quality variables are reported in Table 3. The variance components in Table 3 suggest that the process variable $x_{1,1}$, which measures the performance of Automated Placers 1 and 2, is a major source of variation in the Board Inspection variable $y_{1,4}$. Table 3 also provides evidence that variation in board quality is transmitted to variation in module quality; for example, notice that a majority of the variation in the Module Inspection variables $y_{2,1}$ and $y_{2,7}$ is due to the Board Assembly stage. Finally, we conclude from Table 3 that most of the variation in the Module Inspection variables $y_{2,2}$ and $y_{2,13}$ is due to the Module Assembly stage.

4.5. Managerial Implications

The variance components in Table 3 provide guidance for allocating process quality improvement resources. For example, if management wishes to reduce variation in Board Inspection variable $y_{1,4}$, they should allocate process improvement resources to Automated Placer 1. To reduce variation in Module Inspection variables $y_{2,2}$ and $y_{2,13}$, on the other hand, management should allocate process improvement resources to Module Assembly. Section 5 of the paper deals with the allocation of process improvement resources in a more systematic fashion through the development of an economic model.

Table 3 Estimated Variance Components of Selected Product Quality Variables*†

Source of Variation	Product Quality Variable $y_{1,1}$	$y_{1,2}$	$y_{1,4}$	$y_{1,11}$	$y_{2,1}$	$y_{2,2}$	$y_{2,7}$	$y_{2,13}$	$y_{2,28}$	$y_{2,29}$
BA‡: Auto. Placer 1 and 2 ($x_{1,1}$)	0.01	0.09	0.30	0.00	0.03	0.01	0.02	0.00	0.21	0.19
BA: Auto. Placer 3 ($x_{1,2}$)	0.01	0.03	0.09	0.01	0.00	0.00	0.00	0.00	0.01	0.03
BA: Unobserved Vars. (ε_{1j}'s)	0.98	0.87	0.61	0.99	0.67	0.26	0.79	0.05	0.19	0.25
Module Assembly (ε_{2j})	—	—	—	—	0.30	0.73	0.19	0.95	0.59	0.53

Notes. *Computed using the estimated parameters and Equation (11). The variance of each product quality variable is equal to 1 since the variables are standardized.
†Authors' calculations. Column sums may not equal 1 due to rounding.
‡BA = Board Assembly.

The variance components in Table 3 provide an upper bound on the amount by which variation reduction at a given stage can improve final product quality. For example, we see from Table 3 that improvements to the Module Assembly stage can provide at most a 30% reduction of variation in Module Inspection variable $y_{2,1}$. (The remaining 70% of the variation in $y_{2,1}$ is due to the Board Assembly stage.) Thus, eliminating variation in the Module Inspection (final product), variables will require improvements to both the Board Assembly and Module Assembly stages. We emphasize that the importance of making improvements to the Board Assembly stage is not evident if the correlation between stages is ignored; this illustrates the need to account for quality linkages across stages in identifying (a) the sources of variation in product quality and (b) opportunities for process and product improvement.

4.6. Implications for Process Monitoring and Control

Our results also have implications for interpreting statistical process-control charts and for identifying stages that are out of control. As noted previously, the results in Table 2 provide evidence that the Board Assembly stage has a substantial effect on the output quality of the Module Assembly stage. This suggests that if the Board Assembly stage goes out of control, the output quality of the Module Assembly stage may be adversely affected, causing conventional process-control charts applied to the Module Inspection measurements to signal that the system is out of control. This implies that conventional control charts applied to the Module Inspection measurements cannot distinguish out-of-control conditions in the Module Assembly stage from out-of-control conditions in the Board Assembly stage. To the extent that knowledge of the out-of-control stages enables the manufacturer to take corrective action more quickly, this underscores the need for process-monitoring procedures that identify the stage (or stages) in the system that have departed from control.

4.7. Accounting for Rework

For the present application, the procedure in §3 estimates the $q \times q$ ($q = 41$) coefficient matrix $\mathbf{B}$ by

$$\widehat{\mathbf{B}} = \begin{pmatrix} 0 & \widehat{\mathbf{B}}_{12} \\ 0 & 0 \end{pmatrix}, \tag{20}$$

where the $\mathbf{0}$s are matrices of zeros and

$$\widehat{\mathbf{B}}_{12} = (\mathbf{Y}_1'\mathbf{Y}_1)^{-1}\mathbf{Y}_1'\mathbf{Y}_2 \tag{21}$$

is the estimator of the $q_1 \times q_2$ ($q_1 = 11$, $q_2 = 30$) coefficient matrix $\mathbf{B}_{12}$. A problem with the estimator in (21) is that boards whose quality measurements fall outside specifications are reworked. For each board that undergoes rework following Board Inspection, the quality measurements gathered at Board Inspection do not correspond to the quality of the board when it enters the Module Assembly stage. Hence, use of (21) will yield a biased and inconsistent estimate of the coefficient matrix $\mathbf{B}_{12}$, since (21) does not account for rework. To account for rework, modify the procedure in §3 as follows: For each reworked board, replace the observations in $\mathbf{Y}_1$ in (21) by the corresponding observations observed following rework. Next, let

$$\mathbf{E}_2 = \mathbf{Y}_2 - \mathbf{Y}_1 \widehat{\mathbf{B}}_{12}, \quad (22)$$

where $\widehat{\mathbf{B}}_{12}$ is given by (21), and again, for each reworked board, the observations in $\mathbf{Y}_1$ are replaced by the corresponding observations observed following rework. Finally, estimate $\boldsymbol{\Sigma}$ by (15) and (16) with $\mathbf{E}_2$ given by (22).

5. Investments in Process Quality Improvement

Sections 2 and 3 discuss the measurement of each stage's contribution to variation in the product quality variables. We now propose an economic approach that uses this knowledge in conjunction with economic criteria to facilitate informed decision making about investments in process quality improvement. Building on the simultaneous-equations model developed in §2 and research on continuous quality improvement and learning, we develop an economic model that links investments in quality improvement to reduced variation in product quality and, in turn, reduced quality costs. The economic model captures the trade-off between investments in quality improvement and quality costs, wherein reductions in quality costs occur at the expense of additional quality-improvement investments (and, vice versa, reductions in quality-improvement investments occur at the expense of additional quality costs). With a view to helping management allocate resources efficiently, we use the model to derive the optimal amount of investment in quality improvement for each stage of the manufacturing system.

The *cost of quality* is the sum of all expenditures associated with (a) ensuring that manufactured units conform to specifications or (b) producing units that do not conform. Specifically, the cost of quality is sum of *prevention costs* (investments in preventing nonconforming units from being produced), the costs of inspecting units to determine if they meet specifications, and the costs of producing nonconforming units. Research and experience shows that the cost of quality is directly related to variation in product quality (Taguchi 1986). Hence, a key focus of quality-improvement programs is reducing variation in product quality.

Realizing reductions in variation requires knowledge of what influences variation as well as appropriate allocation of resources to support variance reduction initiatives. With this in mind, we augment the simultaneous-equations model developed in §2 with research by Moskowitz et al. (2001) and Plante (2000) on allocating resources to support quality improvement. Moskowitz et al. develop a dynamic model for resource allocation decisions that integrates learning, variation, and the cost of quality. The fundamental premise of their model is that reducing variation occurs via learning, and investment (resource allocation) decisions are guided by the trade-off between investments in learning and the cost of quality. The models of Moskowitz et al. (2001) and Plante (2000) are designed to guide decision making about quality-improvement targets for suppliers. By viewing each stage of the manufacturing system as an internal supplier, we adapt their models to our multistage setting. In the remainder of this section, we (1) model the cost of quality as a function of variation, (2) present a model of the relationship between variation and learning, (3) combine these models into a cost model that describes the trade off between investments in learning and the cost of quality, and (4) use this cost model to derive the optimal investment in learning for each stage of the manufacturing system.

5.1. Impact of Variation on the Cost of Quality

Here we model the cost of quality as a function of variation in the product quality variables. The cost models proposed by Taguchi (1986) are widely used for this purpose in both research and practice. Following Taguchi (1986), we model the *per unit cost of quality* associated with each product quality variable by a quadratic loss function (see Sullivan 1984), which is a function of the squared deviation of the product quality variable from the nominal performance or target value. We assume that each product quality variable is *centered* in the sense that its mean is equal to the target value. (If the product quality variable is not centered, centering can be achieved by using methodologies such as those proposed by Derringer and Suich

1980, Barton and Tsui 1991, and Plante 1999, 2001.) The *Taguchi loss* for the jth product quality variable observed at the inspection following stage i can be written as $c_{ij}y_{i,j}^2$, where c_{ij} is a cost coefficient that can be estimated as described by Sullivan (1984). Taking the expectation of the Taguchi loss and using the fact that $y_{i,j}$ has zero mean, we obtain the *expected per unit cost of quality*:

$$c_{ij}\,\mathrm{Var}(y_{i,j}). \tag{23}$$

The *total expected per unit cost of quality* for all product quality variables combined is then

$$E(TC) = \sum_{i=1}^{M}\sum_{j=1}^{q_i} c_{ij}\,\mathrm{Var}(y_{i,j}). \tag{24}$$

5.2. Impact of Learning on Variation

Equation (24) describes the effect of variation on the total expected cost of quality. We now consider how the manufacturer can reduce the total expected cost of quality by affecting variation. Section 2 of this paper quantifies variation and identifies its sources. What remains is a model for assessing the impact of investments in learning on variation. To this end, we now consider the learning model of Moskowitz et al. (2001).

The operations management literature suggests that quality improvement is achieved via learning. Learning consists of both *autonomous learning* (learning-by-doing) and *induced learning* (Dutton and Thomas 1984, Lapré et al. 2000, Ittner et al. 2001). Induced learning—learning that occurs as a result of proactive investment in quality improvement—results from a deliberate effort to improve process quality via enhancements in technology, where technology is broadly defined as an integrated process for creating and implementing knowledge. Induced learning can be divided into three parts: (a) engineering learning resulting in improved designs for parts, assemblies, and processes, (b) manufacturing learning resulting in reduced operator errors and ill-advised process adjustments via improved methods and training, and (c) management learning resulting in improvements in organizational structure, better coordination of engineering, manufacturing and service functions, and increased focus on quality and reliability (Kneip 1965). We may view induced learning as a "time machine" in the sense that it accelerates quality improvement to a point that would be achieved, in due time, via autonomous learning. Indeed, it was observations from several years of experience working with manufacturing organizations that led Moskowitz et al. (2001) to consider the impact of induced learning on quality improvement. In one instance, a large consumer electronics manufacturer had required nine months from startup to reach mature product quality. Through investments in induced learning, this time was reduced to two weeks.

In modeling the effect of learning on variation in product quality, Moskowitz et al. (2001) provide an expression for the variance of product quality variables as a function of learning. Here we adapt their approach to our correlated-stage setting. Let λ_i (a decision variable) denote the rate of learning at stage i, and define $f_i = t_i/(t_i - 1)$, where $t_i \geq 2$ represents the amount of experience with stage i (see Plante 2000). t_i could be cumulative production output (i.e., the cumulative number of units produced) or cumulative time. From our specification that $t_i \geq 2$, it follows that f_i ranges from 2 (virtually no experience) to 1 (considerable experience). Let $\mathrm{Var}_r(y_{i,j}; f_1^{-\lambda_1}, \ldots, f_i^{-\lambda_i})$ denote the variance of $y_{i,j}$ as a function of learning in stages 1 through i. Then, applying Moskowitz et al. (2001) approach to our correlated-stage model, we have for the example in Figure 2B:

$$\mathrm{Var}_r(y_{1,1}; f_1^{-\lambda_1}) = f_1^{-\lambda_1}(\alpha_{111}^2\,\mathrm{Var}(x_{1,1}) + \sigma_{11}^2) \tag{25}$$

$$\begin{aligned}\mathrm{Var}_r(y_{2,1}; f_1^{-\lambda_1}, f_2^{-\lambda_2}) &= f_1^{-\lambda_1}(\alpha_{111}^2\beta_{1121}^2\,\mathrm{Var}(x_{1,1}) + \beta_{1121}^2\sigma_{11}^2)\\ &\quad + f_2^{-\lambda_2}(\alpha_{211}^2\,\mathrm{Var}(x_{2,1}) + \sigma_{21}^2).\end{aligned} \tag{26}$$

Equation (25) specifies that a positive rate of learning at Stage 1 ($\lambda_1 > 0$) leads to reduced variation in $y_{1,1}$, the product quality variable observed immediately following Stage 1, except in the extreme case of considerable experience ($t_1 = \infty$ and $f_1 = 1$). From (26), we also see that a positive learning rate at Stage 1 leads to reduced variation in $y_{2,1}$, the product quality variable observed following Stage 2 (provided that

$\beta_{1121} \neq 0$ and $f_1 > 1$). Finally, from (26), a positive learning rate at Stage 2 leads to reduced variation in $y_{2,1}$ (but not $y_{1,1}$). In general, the reduction in variation increases as the learning rate λ_i increases.

5.3. Total Expected Costs

The previous two subsections establish a link between learning and the total expected cost of quality. Induced learning leads to reduced variation and in turn, via (24), a lower total expected cost of quality. As mentioned previously, induced learning is not free but rather requires investing in learning. We now discuss the link between the learning rate λ_i and investments in learning. A reasonable assumption is that investments in learning are directly related to the learning rate. We model I_i, the investment (per unit) required for stage i to achieve a learning rate of λ_i, as a quadratic function of the learning rate:

$$I_i = w_i \lambda_i^2 \qquad (i = 1, 2, \ldots, M), \tag{27}$$

where w_i (> 0) is a cost coefficient that can be estimated as described by Moskowitz et al. (2001). This investment function (27) in combination with the learning model illustrated in (25) and (26) imply that investments in learning face diminishing marginal returns with respect to (a) variation and (b) the total expected cost of quality (24). Moskowitz et al. (2001) compare linear, quadratic, and exponential investment functions, and find that the quadratic investment function in (27) yields investment allocations that approximate the common practice of setting quality improvement goals at a 20% reduction in variation.

Augmenting the total expected per unit cost of quality (24) with investments in learning (27) and the reduction in variation resulting from learning, we get

$$E(TC') = \sum_{i=1}^{M} \left[w_i \lambda_i^2 + \sum_{j=1}^{q_i} c_{ij} \operatorname{Var}_r(y_{i,j}; f_1^{-\lambda_1}, \ldots, f_i^{-\lambda_i}) \right]. \tag{28}$$

This cost model (28) captures the trade-off between total expected quality costs and investments in learning. According to the model, quality-cost reductions occur at the expense of additional investment in learning, and vice versa, reductions in learning investments occur at the expense of increased quality costs.

5.4. Optimal Investments in Learning

The learning rates $\lambda_1, \lambda_2, \ldots, \lambda_M$ that minimize total expected cost (28) are a solution to the first-order conditions

$$\begin{aligned} \frac{\partial E(TC')}{\partial \lambda_i} &= 2w_i\lambda_i + \sum_{j=1}^{q_i} c_{ij} \operatorname{Var}_r^{(i)}(y_{i,j}) \\ &\quad + \sum_{k \in U_i} \sum_{l=1}^{q_k} c_{kl} \operatorname{Var}_r^{(i)}(y_{k,l}) \\ &= 0, \quad i = 1, 2, \ldots, M, \end{aligned} \tag{29}$$

where U_i is the set of stage i's successors and $\operatorname{Var}_r^{(i)}(y_{k,l})$ is the first partial derivative of $\operatorname{Var}_r(y_{k,l}; f_1^{-\lambda_1}, \ldots, f_k^{-\lambda_k})$ with respect to λ_i. It can be shown that the Hessian matrix of (28) is positive definite; hence, (28) is a convex function of $\lambda_1, \lambda_2, \ldots, \lambda_M$ and there exists a unique solution to the first-order conditions (29). Although they cannot be found directly, the values of $\lambda_1, \lambda_2, \ldots, \lambda_M$ that solve (29) can be found by using the Newton-Raphson method. Letting λ_i^* denote the optimal learning rate for stage i, the optimal level of *investment* for stage i is given by (27) with $\lambda_i = \lambda_i^*$ ($i = 1, 2, \ldots, M$).

Consider for illustration the example in Figure 2B. The learning rates λ_1 and λ_2 that minimize total expected cost (28) are the unique solution to the first-order conditions

$$\begin{aligned} \frac{\partial E(TC')}{\partial \lambda_1} &= 2w_1\lambda_1 - f_1^{-\lambda_1}\ln(f_1) \\ &\quad \times [c_{11}(\alpha_{111}^2 \operatorname{Var}(x_{1,1}) + \sigma_{11}^2) + c_{21}(\alpha_{111}^2 \beta_{1121}^2 \\ &\quad \times \operatorname{Var}(x_{1,1}) + \beta_{1121}^2 \sigma_{11}^2)] = 0, \end{aligned} \tag{30a}$$

$$\begin{aligned} \frac{\partial E(TC')}{\partial \lambda_2} &= 2w_2\lambda_2 - f_2^{-\lambda_2}\ln(f_2) \\ &\quad \times [c_{21}(\alpha_{211}^2 \operatorname{Var}(x_{2,1}) + \sigma_{21}^2)] = 0. \end{aligned} \tag{30b}$$

The solution can be found by using the Newton-Raphson method.

6. Discussion

This study proposes and validates a procedure for measuring the impact of each stage's performance on the output quality of subsequent stages, including the quality of the final product. The procedure builds on the precedence ordering of the stages in the system and uses the information provided by correlations between the product quality measurements across stages. The procedure is easy to implement and is applicable to manufacturing systems that contain large numbers of stages.[5]

From a managerial perspective, the procedure is appealing because it identifies stages where process improvements have the potential to yield significant improvements in product quality. Moreover, the variance components estimated by the procedure can be monitored over time in order to (a) detect changes in process quality and (b) track the effectiveness of quality-improvement (i.e., variation reduction) efforts. Finally, the network representation of the model is easy to understand and computer executable.

By quantifying the impact of each stage's performance on variation in product quality, our procedure provides a foundation for evaluating process quality improvement strategies. Combining knowledge of each stage's impact on variation in product quality with research on continuous quality improvement and learning led to the development of our economic approach for allocating investments in quality improvement. Our economic approach finds the optimal amount of investment in quality improvement for each stage of the manufacturing system, providing guidance to management on investing in quality improvement.

The findings from our empirical analysis of the production line data carry a number of implications for the management of quality-improvement activities. For example, as discussed in §4.5, quality linkages across stages need to be accounted for in identifying the sources of variability in product quality. A corollary of this is that manufacturers who ignore quality linkages may make suboptimal investments in process quality improvement. Thus, our results demonstrate the need for manufacturers to account for quality linkages in making decisions about investments in quality improvement. As discussed in §4.6, our finding of substantial correlations between the product quality variables across stages provides empirical evidence that conventional process-monitoring procedures are not appropriate for identifying stages that are out of control. This emphasizes the need for manufacturers to use monitoring procedures that are designed for multistage manufacturing systems.

We hope this study facilitates more effective investments in process quality improvement. We also hope this work provides a rigorous and coherent framework for future research on quality improvement in multistage manufacturing systems. An important avenue for future research is extending our model and procedure to allow for scrapping of nonconforming units prior to the final inspection.

[5] We have implemented the procedure using the matrix language SAS/IML (SAS Institute 1990).

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